FST4W on the HF Bands: Why - What to expect - Equipment - Results.

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Summary

A greater use of FST4W on the HF bands could bring several benefits, the marginal sensitivity increase of 1.4 dB over WSPR for the 120-second variant probably being the least valuable. Other potential benefits are greater tolerance to Doppler spread and the significant increases in sensitivity from the longer sequences. While ionospheric Doppler spread will limit the use of the longer sequences, maximum utility of FST4W can come from careful attention to minimising spectral spread within transmitters and receivers. These are areas where the amateur can make improvements. As the protocol measures spectral width it provides the essential tool to both gauge equipment performance and also bring insights into propagation that SNR alone does not provide

1. Introduction - Why HF band FST4W?

The FST4W (and FST4) digital protocols were introduced in WSJT-X 2.3.0 together with a Quick-Start guide[1]. Table 1 summarises the protocol's basic parameters. While designed specifically for the LF and MF bands FST4W has several advantages over WSPR that could prove beneficial on the HF bands. These are:

- Option of four sequence lengths, designated in seconds as FST4W-120, -300, -900 and -1800. While sensitivity increases with sequence length, tolerance to spectral spread decreases.
- Better sensitivity, e.g. the FST4W-120 SNR threshold is about 1.4 dB lower than WSPR.
- Greater tolerance to Doppler spread, e.g. for a signal with a Doppler shift of 2 Hz FST4W-120 will decode at -24 dB SNR while WSPR requires -17 dB SNR.
- Optional measurement of the spectral spread. With WSPR we cannot say whether failure to decode a spot was due to inadequate SNR or excess spectral spread. FST4W gives us insights into spectral spread that lets us determine the likely cause. Spectral spread may also show patterns related to propagation, e.g. at band opening and closing, dusk and dawn, during geomagnetic disturbances and their aftermath that may be of interest.

Despite these potential advantages over WSPR to date the uptake of FST4W on the HF bands has been minimal. Users of WSJT-X on receive have to preselect a single sequence length to decode, thereby missing reception of other sequence lengths, let alone WSPR in the same band. In a substantial upgrade to WsprDaemon[2] 3.0 author Rob Robinett has enabled decode of all requested FST4W sequence lengths as well as WSPR on each selected band. The aim is that by enabling simultaneous multiband, multimode decoding this will encourage more amateurs to transmit FST4W. A real-time list of WsprDaemon users that shows those with FST4W decode enabled is available online[2].

Parameter	-120	-300	-900	-1800
SNR threshold (dB)	-32.8	-36.8	-41.7	-44.8
Symbol length (s)	0.683	1.792	5.547	11.200
Tone spacing (Hz)	1.46	0.56	0.18	0.089
Occupied bandwidth (Hz)	5.9	2.2	0.72	0.36
Measured spectral width (mHz)	5.3	2.14	0.74	0.35

Table 1. Basic parameters for the FST4W digital protocol for the four sequence lengths together with the measured baseband spectral width from jt9. SNR threshold is in dB in a 2500 Hz bandwidth. Except for measured spectral width and an SNR threshold of -31.4 dB WSPR-2 has the same parameters as FST4W-120.

This paper is organised as follows. Section 2 outlines what an FST4W user on the HF bands may expect, drawing on the protocol modelling tool *fst4sim* included in the WSJT-X package. Section 3 reinforces the message in the Franke et al. Quick-Start Guide that receiver and transmitter oscillator drift and short-term frequency jitter need to be considered carefully within an overall spectral spread budget for each sequence length. The examples in section 4 concentrate on the 14 MHz band, ranging from surface wave over tens of km to single and multiple hop ionospheric F layer propagation when the earth's geomagnetic field is quiet and active. The paper ends with a discussion of these and other applications for this underutilized digital protocol having clear potential on HF bands as a propagation reporter and investigative tool.

2. What to expect?

Figure 1 summarises two of the advantages of FST4W-120 over WSPR. First, at spectral spreads of less than 0.2 Hz there is 1–1.4 dB better sensitivity. Second, above a spectral spread of 2 Hz the rate of rise of required SNR is less for FST4W-120 than for WSPR. At a spectral spread of 3 Hz WSPR will not decode whatever the SNR.

The potential advantage of much lower decode thresholds for the longer sequences, Figure 2, are realised in practice at LF and MF where spectral spreads due to propagation and equipment are lower than on the HF bands. The practical pointers in Section 3 on equipment requirements and the examples of different propagation conditions in section 4 illustrate what can be achieved on 14 MHz. Suffice to say here that the -1800 sequence length will likely only be of use in few cases at upper HF, e.g. given surface wave propagation and GPSDO oscillators at transmitter and receiver.

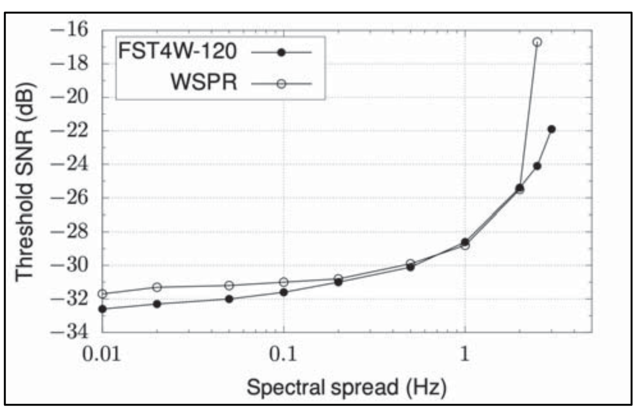


Figure 1. A comparison of the decode threshold with spectral spread for WSPR and FST4W-120. Acknowledgement: Based on figure provided by Steve Franke K9AN.

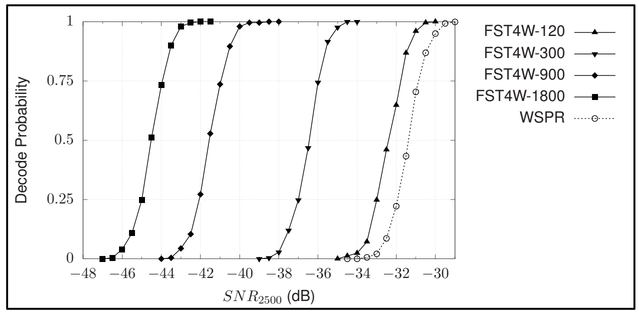


Figure 2. How decode probability varies with SNR near the threshold for FST4W-120 to -1800 in comparison with WSPR. Acknowledgement: figure provided by Steve Franke K9AN.

2.1 Spectral spread and spectral width

It is important to address a matter of likely confusion between spectral spread and spectral width. In this paper spectral spread in an 'input' parameter, e.g. as *fdop* in the fst4sim command line. In contrast spectral width is an 'output' parameter from the *jt9* executable.

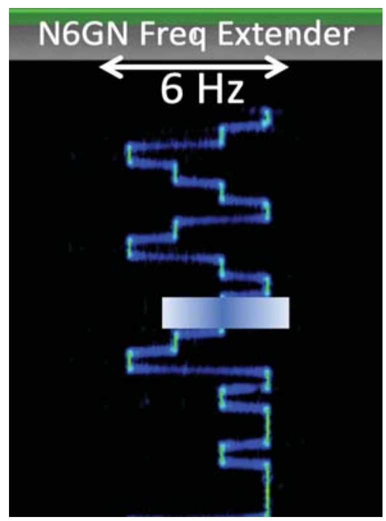


Figure 3. Waterfall display of the spectrum of the 33rd harmonic of a 14 MHz local FST4W-120 transmission from on ANAN SDR at N6GN using a KiwiSDR with GPSDO and a homebrew frequency extender. The image shows the smooth transitions between tones and the fraction of a tone period they span. The rectangle at the centre represents additional spectral spreading.

For a simple pulse of constant amplitude and frequency the bandwidth-time product (BT) is 1, that is, a 2 s long pulse will have a bandwidth of 0.5 Hz. However, for the Gaussian Frequency Shift Keying modulation scheme adopted for FST4W BT is less than 1, set by the bandwidth of the Gaussian filter. For BT=1 the expected spectral width of a 109.3 second transmission (the actual duration for -120) would be 9.15 mHz, i.e. 1/109.3 Hz. From *jt9* the measured spectral width of WSJT-X baseband audio was 5.3

mHz, suggesting a Gaussian filter with a BT~0.6, comparable to that of Bluetooth at 0.5).

The important point is that the *jt9* output spectral width is for a single tone, and not the occupied bandwidth of the received signal spanning four tones. Figure 3, a waterfall spectrum from author Glenn Elmore of the 33rd harmonic of a locally generated 14 MHz FST4W-120 transmission, shows the four-tone G-FSK with smooth (Gaussian) transitions between tones. A rectangle, added to the graphic, illustrates how a spectral spreading increase encroaches on the other frequencies, causing inter-symbol interference while also reducing SNR.

2.2 Uses of the fst4sim simulator

While the information in Figures 1 and 2 give us a good starting point in setting expectations for FST4W the *fst4sim* simulator that is available as a command line tool in WSJT-X lets us explore other potential pitfalls and unexpected outcomes. The command line is:

fst4sim "message" TR f0 DT fdop del nfiles SNR F
where:

message - callsign, locator and power to encode, e.g. "G3ZIL IO90 23"

TR - sequence length in seconds, values for FST4W being 120, 300, 900 and 1800.

f0 - baseband frequency, our examples use 1500 Hz.

DT - time offset in seconds between tx and rx.

fdop - notionally the (ionospheric) Doppler spread, but in this work we take this value as the total *spectral* spread that includes spreading due to the tx and rx. fst4sim uses an embedded version of an ITU channel simulator[3] in turn based on the Watterson[4] model.

del - the differential path (multipath) delay in milliseconds. We have not explored this parameter and use the default of 1 ms; in all cases the differential delay is much shorter than the FST4W symbol lengths.

nfiles - number of simulations to run.

SNR - the input SNR, which we will compare with the model output. In all cases we have used -15 dB.

F - a flag to indicate FST4W mode.

The simulator generates number *nfiles* of wav files of the selected length that the *jt9* command line executable can decode. By including an empty file named *plotspec* in the directory in which *jt9* is run the spectral width will also be calculated and displayed in a WSJT-X window.

2.2.1 Effect of time offset

While rarely a problem these days (a check on the time offset of 1000 WSPR spots on 20 m showed a lower decile of 0.024 s and an upper decile of 1.09 s) larger time offset can occur, e.g. using a remote KiwiSDR via a browser. Figure 4 shows the output SNR and output spectral width when fdop=0 for FST4W-300. Between time offsets of -1.0 s to +2.0 s both SNR and spectral width values are well behaved. However, while still decoding the data, biased values are reported out between -1.5 s and -1.0 s and between +2.0 s and +2.4 s, with SNR biased low and spectral width biased high. Other sequence lengths show the same general behaviour over very similar time offsets.

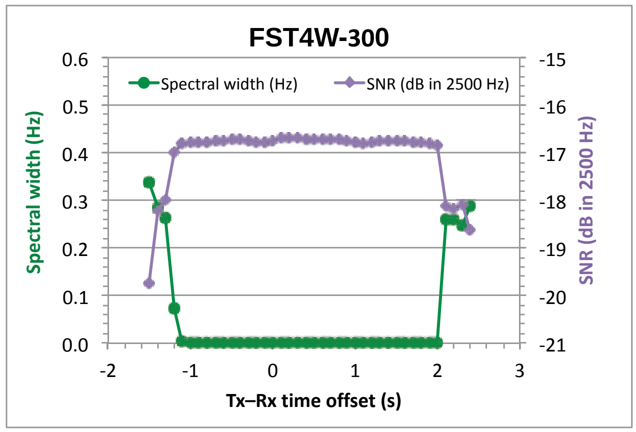


Figure 4. Variation of SNR and spectral with from fst4sim with time offset between transmitter and receiver for -300, showing biased values at each extreme but before no decode is possible.

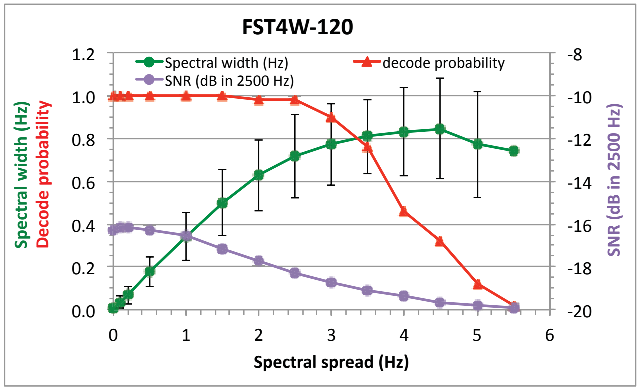
Closely related to time offset, measurements of the effect of changing a KiwiSDR's *abuf* setting for browser connections are described in section 3.

2.2.2 Calculated spectral width, decode probability and SNR with spectral spread

For these simulations *nfiles* was set at 50 for each value of *fdop* and the mean and standard deviation of the computed set of SNRs and spectral widths calculated. The probability of decode at each *fdop* was estimated from how many of the 50 wav files were decoded. Figure 5 shows the variation of calculated SNR, spectral width and decode probability for the -120 and -300 sequence lengths vary with spectral spread.

We see that:

- At zero *fdop* the decoded SNR is over 1 dB down on the -15 dB input *SNR* for an unknown reason.
- SNR becomes further biased low as *fdop* increases, reaching -2 dB even with probability of decode



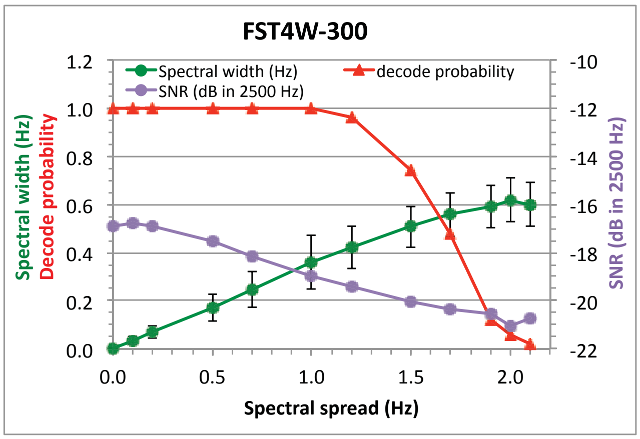


Figure 5. Variation of spectral width, decode probability and SNR with spectral spread from 50 runs of the fst4sim model at each value of Doppler spread for -120 upper, and -300 lower. The bars on spectral width are at +/-2.5 times the standard deviation, that is, 99% of the values lie within the interval.

above 95%. The bias is greater for -300 than for -120.

- The *jt9* output spectral width values show a linear trend with input spectral spread until the decode probability starts to drop. The output spectral width ends up being biased low as it is only those transmissions with (statistically) lower spectral widths that are decoded and therefore included in the measurements.

In summary, these graphs alert us to expect instances of SNR and spectral width biased low as the input spectral spread increases. They also help set expectations for probability of decode.

3. Equipment

While the ionosphere is probably the most variable contributor to spectral spread on the HF bands, as highlighted in the FST4 Quick-Start Guide[1], it may not necessarily be the largest. Spectral spread inevitably arises, to some degree, at a transmitter and receiver from oscillator drift and shorter-term jitter

It is not a trivial task to characterize the spectral spread of transmitters and receivers. A key enabler for this study was to use external GPSDO frequency control of four reference transmitters and receivers, namely:

- Transmitter: ANAN 'Angelia' SDR transceiver at N6GN, Fort Collins, Co.
- Remote receiver: KiwiSDR 21 km from N6GN.
- Transmitter: ANAN-10 at K6PZB, Graton, Ca.
- Receiver: KiwiSDR at WB7ABP/K, Santa Rosa, Ca., 10 km from K6PZB.

These systems, with master oscillators phase-locked to GPS, provided the basis for calculating the spectral spread of other KiwiSDRs using their standard GPS aiding algorithm. This algorithm is best described as intermittent correction rather than frequency-lock, let alone phase-lock.

As FST4W provides us with estimates of overall spectral spread from transmitter to receiver via the propagation path knowing the transmitter and receiver spectral spread estimates we can estimate the remaining spread due to the propagation path, e.g. for one- and two-hop F-layer propagation. Full details of these calculations are in the Annex while example equipment spectral spreads are described below.

3.1 Spectral spread of GPSDO transmitters and receivers.

We could not measure the spectral spreading of a GPSDO receiver or transmitter alone, only in combination over a 21 km surface wave path. The histogram in Figure 6 shows the distribution of spectral width output by *jt9* from 543 spots between 19–23 July 2022 over a completely line-of-sight path. The peak at 0.00625 Hz is almost double the audio baseband width of 0.0035 Hz.

As there were identical GPSDOs at each end, we will assume equal contributions to spreading. This positive-only, unimodal distribution with an extended right tail lends itself a Gamma distribution fit, the red line. From the Gamma distribution we can split equally into spectral width estimates for the GPSDO transmitter and receiver alone (see Annex for the methods used).

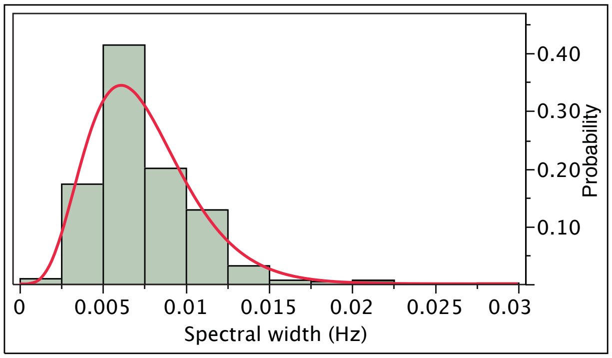


Figure 6. Histogram of the distribution of FST4W-300 spectral width determinations with GPSDO transmitter N6GN and GPSDO receiver N6GN/K over a 21 km line of sight path, from 543 spots 19–23 July 2022. In red is a fitted Gamma distribution, shape=5.62 and scale=0.00132.

3.2 Some issues when not using a GPSDO

In practice, many potential users of FST4W on the HF bands may not have access to GPSDOs. Depending on the characteristics of the hardware there may be other spectral spread mechanisms in equipment commonly used for WSPR, such as:

- A transceiver used for sending and receiving FST4W, even at 5 W output, may reach an elevated temperature such that at the end of the transmit cycle it will take many minutes for its (quite likely TCXO) master oscillator to settle to a sufficiently stable frequency for -300 and longer sequences.
- The innovative approach to mapping audio to RF frequency, emulation of the Gaussian slide between tones, and the tone spacing on transmit in the QRP Labs QDX digital modes transceiver[5] merits further evaluation. Although the receive side is well suited for FST4W to -900 the signal generated on transmit appears to be nowhere nearly as good at the time of writing (firmware 1.04).

Because of the ubiquity of KiwiSDRs with standard GPS aiding two spectral spread issues are summarised below followed by an assessment of the low-cost QDX with its TCXO.

3.2.1 KiwiSDR short-term frequency stability

While perfectly acceptable for WSPR and FST4W-120 the short-term frequency stability of the KiwiSDR with its standard GPS aiding is almost certainly a limitation for the longer FST4W sequences at least on the upper HF bands. Figure 7 shows two measures of short-term frequency variations.

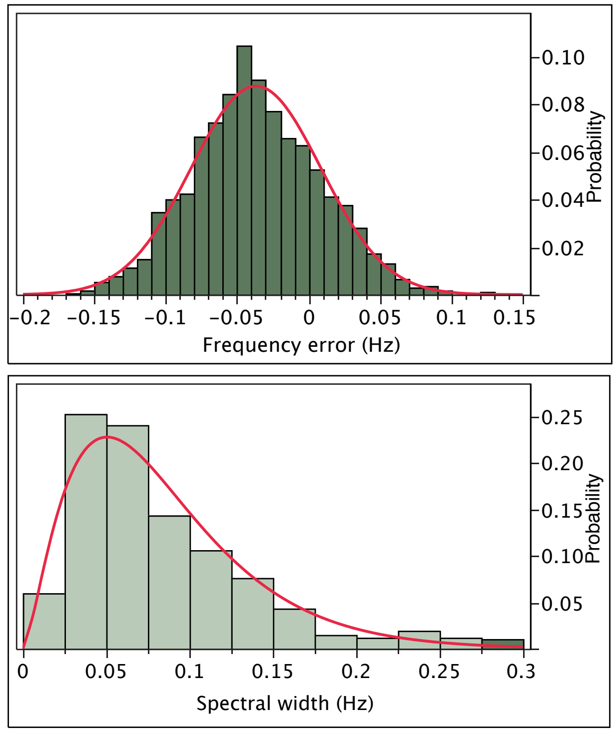


Figure 7. Two measures of short-term frequency variation in two KiwiSDRs. Upper: Histogram and fitted Gaussian distribution at G3ZIL of frequency error at 10 MHz at one-second intervals. Lower: Spectral width from jt9 at KPH from FST4W-120 GPSDO transmissions from K6PZB over a 38 km path.

The upper histogram is of frequency error at 10 MHz against a 10 MHz GPSDO from measurements one second apart using the frequency analysis tool in fldigi on a captured wav file. The red curve is a fitted Gaussian distribution with a mean error of -0.037 Hz and a standard deviation of 0.046 Hz.

The lower histogram is spectral width from a standard KiwiSDR at KPH, Point Reyes, Ca. of transmissions from a GPSDO transmitter at K6PZB 38 km distant for FST4W-120. While this includes the spectral spreading at the transmitter, from Figure 6 this would be less than 10% of the total. The median spectral spread is 0.068 Hz. This is of the same order as the standard deviation of one-second frequency errors measured at G3ZIL.

3.2.2 KiwiSDR browser connection abuf setting

If accessing a KiwiSDR via a browser reducing the buffer duration by specifying abuf=0.5 in the url, e.g. http://10.0.1.234:8073/?abuf=0.5 will reduce spectral spreading from that with the default value and reduce the time offset, Table 2. Those reductions will increase the margins for spectral spreading and DT for ionospheric paths and transmitters with time offsets. Note that setting a long buffer time is counterproductive; the result is a large spectral spread and a much reduced decode probability likely associated with the higher DT (section 2.2.1).

Parameter	**Default**	**0.5**	**1.0**	**1.75**
Median (Hz)	0.088	0.057	0.085	0.667
% SW/TS	6	4	6	46
% decode	100	100	100	23
DT (s)	1.6	1.1	1.5	1.9

Table 2. Median spectral width in Hz and as a fraction of the Tone Spacing together with the probability of decode and the time offset for FST4W-120 from a KiwiSDR via a browser for different values, in seconds, for the buffer length abuf.

3.2.3 QDX receive short-term stability

The QRP Labs QDX is a low cost digital modes transceiver with a TCXO. On receive it has an impressively low standard deviation of frequency error of 0.008 Hz at 10 MHz against a GPSDO and the -2.04 Hz mean error measured at G3ZIL could easily be accounted for in the user-set calibration. In a test at N6GN the measured spectral width receiving FST4W-300 from a GPSDO transmitter was 0.0198 Hz.

At G3ZIL the receive frequency shift after two cycles of transmitting FST4W-300 was an initial 0.75 Hz, returning toward the pre-transmit value with a time constant of about 230 s, but with a 0.2 Hz shift.

These are very encouraging figures on receive, however, the innovative transmitter does require further study.

4. Results

4.1 Main findings

The main findings of this study are summarised in Figure 8. Each column shows the spectral spread 'budget' out to where the *fst4sim* decode probability has dropped to 10% for each sequence length. Each column has three sources of spectral spread: transmitter, receiver and propagation path. The example transmitter and receiver spectral spreads have been derived from the measurements outlined in section 3.

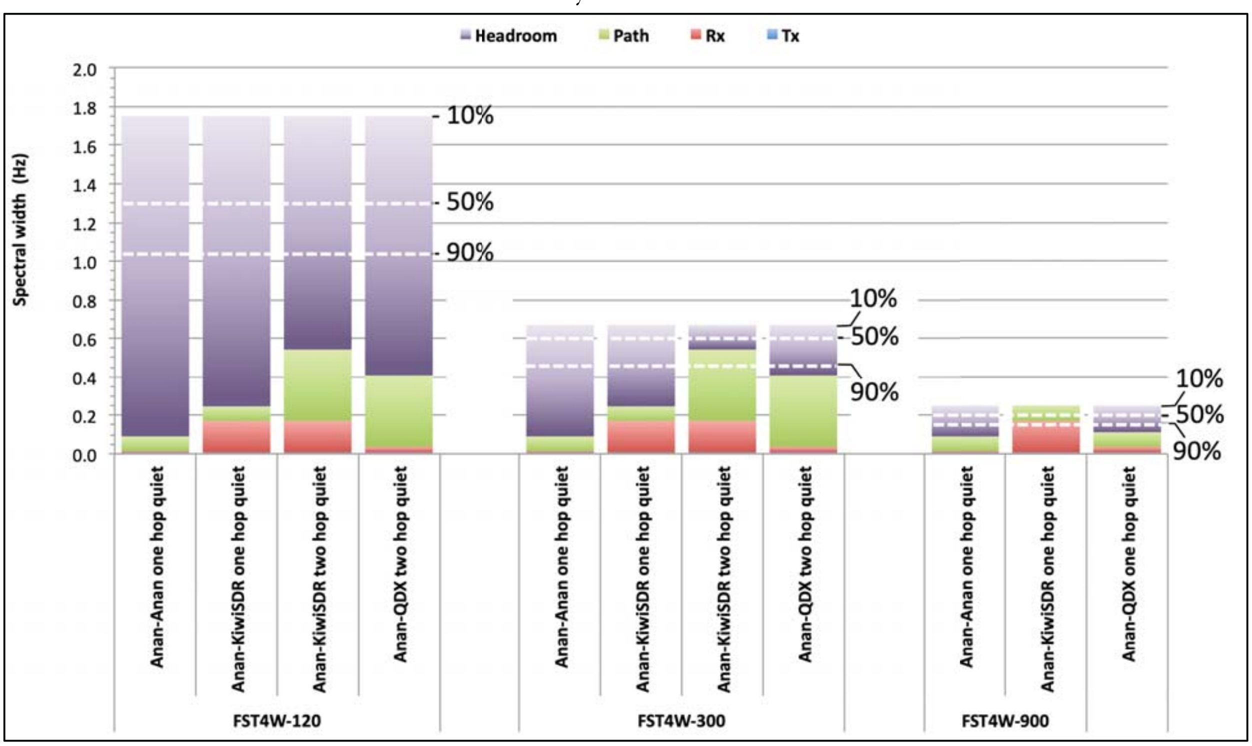


Figure 8. Spectral spread budgets for FST4W-120 to -900 for example combinations of transmitter, receiver and ionospheric propagation on the 20 m band. The upper limit for each column is the spectral spread at which there is only a 10% probability of decoding the signal. Also marked are the limits for 50% and 90% decode probability. While these lines remain in the purple 'headroom' area decode should be possible for quiet conditions (Kp <3).

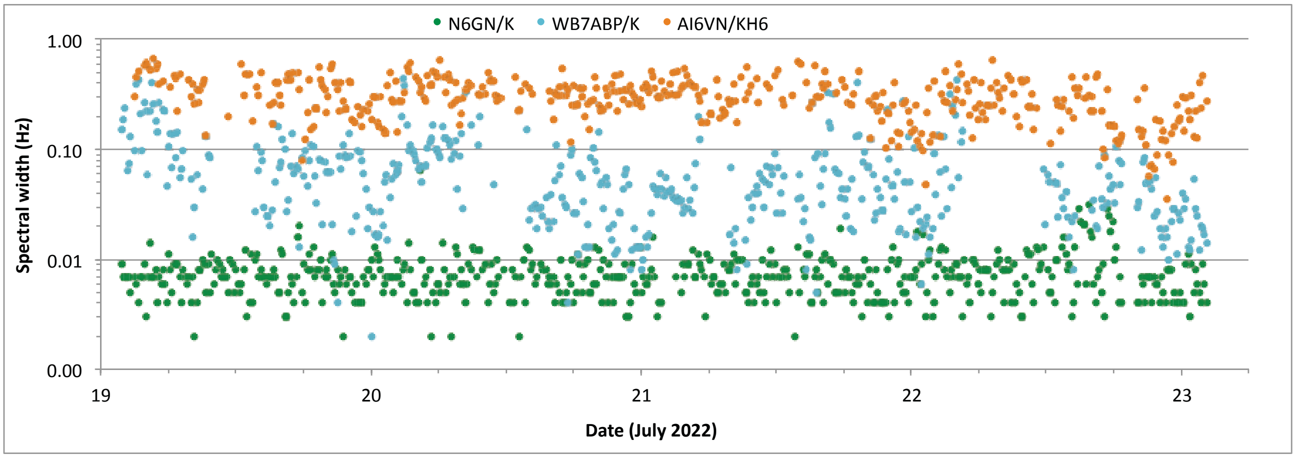


Figure 9. Time series of the spectral width on a log scale as output by jt9 from FST4W-300 transmissions by N6GN for three different propagation paths to: (green) N6GN/K a KiwiSDR with GPSDO at 21 km; (cyan) WB7ABP/K a KiwiSDR with GPSDO at 1558 km; (orange) AI6VN/KH6 a KiwiSDR with standard GPS aiding at 5291 km.

The spectral spreads are at the 90% level, that is, based on the Gamma function modelling, these values would only be exceeded 10% of the time. The propagation path spreads, for one- and two-hop, are from the measurements described later in this section. These were observed on 20 m and an assumption of acceptable extrapolation is made when considered for other bands

In all bar two of the examples (-300 and -900, ANAN and standard KiwiSDR two-hop) there is, to a greater or lesser extent, some headroom between the ionospheric component and the 90% probability level. That is, if the ionospheric spectral spread increases the decode probability will remain above 90%.

On 20 m for -300 with standard KiwiSDR and two-hop, even with a quiet ionosphere, the spectral spread means that the decode probability has already fallen to below 90%. For -900 with standard KiwiSDR and one-hop, the spread budget at 90% probability has already been used by the receiver (and a minor part by the transmitter). Few decodes would be expected. For this reason we have not considered the -1800 mode with half the spectral spread budget of -900. Even fewer decodes would be expected for the higher frequency bands.

From our measurements the QDX receiver with its standard TCXO would be suitable for receiving in -900 one-hop situations on its top band of 20 m. A KiwiSDR with an external TCXO or GPSDO would also be suitable for -900. Arising from this study the KiwiSDR designer has provided an option for updating the oscillator correction only during a gap in FST4W transmissions.

4.2 Spectral widths over contrasting paths

Figure 9 shows a time series of spectral width from *jt9* over four days in July 2022 for transmissions of FST4W-300 from N6GN. The paths were 20 km, 1558 km and 5291 km respectively. All three are at mid-latitudes. The log scale is necessary to show the two orders of magnitude difference between the 20 km and 5291 km paths.

Gaps in reception at WB7ABP/K were, with day-to-day variability, grouped around 0600–1200 UTC when the MUF over the propagation path was at its lowest. The reception pattern at AI6VN/KH6 was different, gaps were more scattered through each day. It is possible that the MUF remained sufficiently high for a range of 5291 km with two-hop propagation.

These spectral width time series include spreading contributions from the transmitter (ANAN with GPSDO, minimal), the receivers (KiwiSDRs with GPSDO at N6GN/K and WB7ABP/K, minimal), and with standard GPS aiding of a KiwiSDR at AI6VN/KH6. Using the method described in the Annex the estimated spectral spread from the ionospheric propagation paths alone were abstracted, with the results shown as model histograms in Figure 10. The spectral spread over the propagation paths from Colorado to California and to Maui are both skewed with a tail to the right, and the mode at the lowest bin. These are characteristics shared by the published[6] Doppler spread over a path from Svalbard

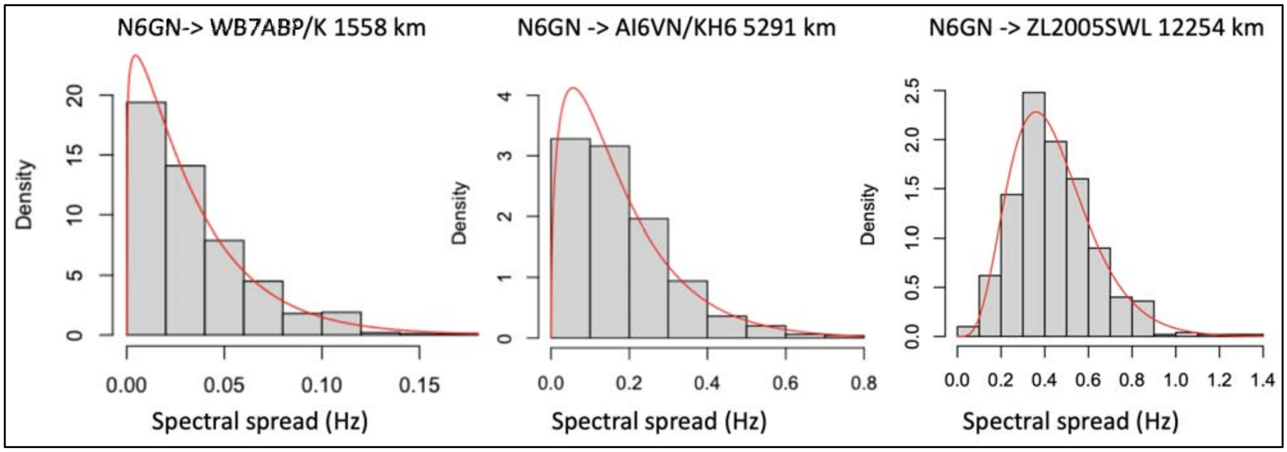


Figure 10. Histograms of spectral spread for the ionospheric path only, derived using the method in the Annex, for three paths from N6GN, Fort Collins, Co., to WB7ABP/K, Santa Rosa, Ca., AI6VN/KH6, Maui, Hi., and ZL2005SWL, North Island, New Zealand. To WB7ABP/K and AI6VN/KH6 transmissions were FST4W-300, and FST4W-120 to ZL2005SWL.

to southern Norway in April 1995 on 9.04 MHz - one of few examples available.

In contrast, the distribution for the 12,254 km trans-equatorial path to short-wave listener ZL2005SWL on the North Island of New Zealand is more symmetrical, with a mode well away from the lowest bin, approaching a Gaussian distribution.

4.3 Some interesting patters of spectral spread

At times, FST4W spectral width estimates appear to contain information related to the physical phenomena underlying the cause of the fluctuations. The WsprDaemon Grafana dashboard for FST4W allows rapid inspection of interesting parts of time series. Figure 11 is one example, from FST4W transmissions from N6GN to WB7ABP/K. There are times where successive, independent measurements, with low scatter, appear to show wave-like behaviour. There was also a steady increase from 00:00 UTC on 20 July, and, from 02:40 to 03:10 an event where four measurements were well above the slow, upward trend. As the MUF dropped, and the band approached closing, there were fewer decodes, the SNR had dropped, and spectral widths were high and scattered.

Finally, there remains a puzzle. The mode of spectral widths of FST4W-120 from the GPSDO transmitter at K6PZB at KPH, 38 km distant on 14 MHz was at 0.085 Hz. At KFS, Half Moon Bay, Ca., 124 km distant, over the example time series in Figure 12, there was just one K6PZB decode with a spectral width below 0.1 Hz. Yet the mode of the spectral width at KFS for FST4W-120 from N6GN 1535 km distant, via the ionosphere, was far lower at 0.06 Hz. What form of propagation was there between K6PZB and KFS to produce such high spectral spreading,

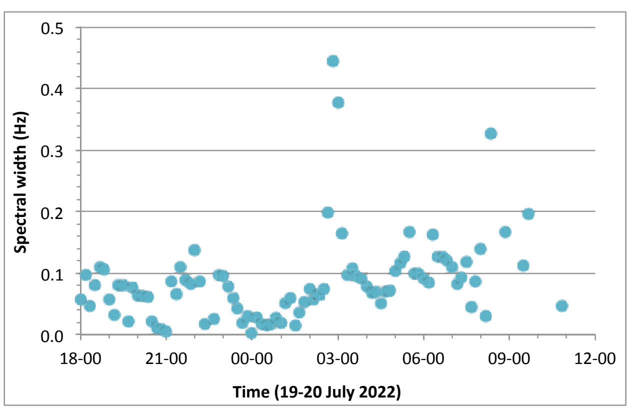


Figure 11. Time series of the spectral width of FST4W-300 transmissions from N6GN received at WB7ABP/K with suggestion of periodic variations, times with very low scatter, an event around 03:00 UTC and a slow rise toward the band closing.

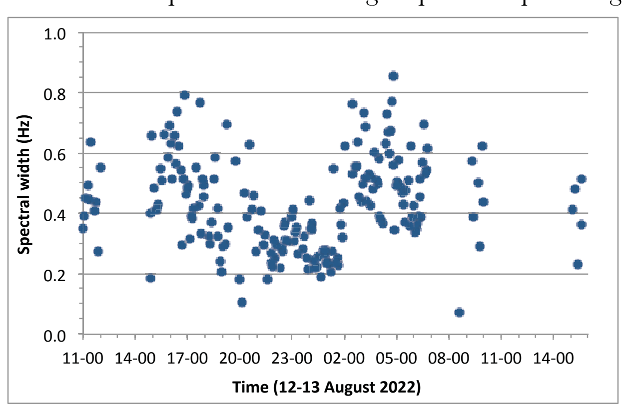


Figure 12. Time series of the spectral width of FST4W-120 transmissions from K6PZB received at KFS using its SW antenna. The separation is 124 km - why the large spectral widths? They were much greater than those from N6GN at 1535 km via the ionosphere at this time.

with SNR between -23 and -10 dB in 2500 Hz bandwidth, and with this temporal pattern? With only SNR, e.g. from WSPR, the question may well not have arisen. As it stands, there may be interesting aspects of surface wave HF propagation yet to be better understood.

4. Discussion and future work

It would seem a missed opportunity if FST4W were to be considered a protocol only suitable for the LF and HF bands. The -120 sequence length has no additional limitations over WSPR; instead it brings the minor benefit of a small increase in sensitivity and a large benefit in its measurement of spectral width. Not only can these measurements spur technical innovation in reducing the spectral spread within transmitters and receivers it can provide additional insight into propagation. It is important to note that sensitivity and spectral width are interlinked - near threshold sensitivities will only be achieved if the spectral spread is low. As we show, equipment-induced spectral spread can exceed that of the ionosphere on HF for one-hop paths.

We have outlined these advantages in this paper together with examples of use and several limitations. In particular, it is likely that the -1800 sequence length will only be of very specialised use on non-ionospheric paths or on lower the HF bands. Furthermore, the -900 sequence length may only be of practical value for one-hop ionospheric paths.

Users of KiwiSDRs and WsprDaemon (who currently report some 26% of all WSPR spots) are well placed to report FST4W transmissions following the WD3.0 upgrade by Rob Robinett. We encourage additional transmissions of FST4W-120 and -300 on the HF bands given this receiver base.

Further studies in the near-term will include:

- Evaluation of the new option on the KiwiSDR to have it alter its clock frequency only during non-data gaps in WSPR or FST4W transmissions.
- In-depth testing to identify the causes of, and potential remediation measures for, significant spectral spreading on transmit of the QDX digital transceiver.
- Adding to the fst4_decode.f90 module within *jt9* to output spectral width for candidate spots that were not decoded. This will help identify where excessive spectral width rather than low SNR was the cause of failure to decode, and by what margin the allowable spectral spread was exceeded.
- What is the propagation mechanism over the 124 km path between K6PZB and KFS and why the large spectral spread compared with a similar but shorter path?
- FST4W into and through the Auroral Oval, working with KiwiSDR installations at Inuvik, Northwest Territories, Canada, and VY0ERC, Eureka, Nunavut, Canada.
- Comparing SNR values for WSPR and FST4W as we have seen significant differences and we do not understand the use of a 12.5*Log10 (sequence length) adjustment in the fst4_decode.f90 program.

Acknowledgement

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Annex. Separating out contributions to spectral spreading from the transmitter, receiver and propagation path using Gamma functions.

This annex describes a practical, statistical method for separating out the spectral spreading from transmitter, receiver and propagation path. It does not attempt to separate out the contributions for a single transmission. The method is applicable where a number of decodes are available over a path and there is some knowledge of the equipment at each end. To begin, experiments over line-of-sight or at least surface wave paths of no more than a few tens of km are needed. The statistical analysis can be done with the R free software for statistical computing.

A Gamma function is an appropriate fit to the data sets observed to date, in that they are positive only, essentially unimodal and with a long right-hand tail. If a distribution is not unimodal there may be reasons, e.g. the data set spans distinctively different propagation conditions. In that case one should attempt to separate, perhaps into two or more time periods, and analyse separately.

Using the equipment and paths in this paper to illustrate the method, the key steps are:

1. Over a line-of-sight or surface wave path of a few 10s of km use a transmitter and receiver with GPDSO master oscillators with WsprDaemon or WSJT-X with empty file plotspec added to obtain spectral widths. Using R or another package plot spectral widths histogram, fit a Gamma distribution, and obtain two parameters: Shape and Scale (note that the mode is at Shape*Scale). Figure A1 (upper) is for N6GN to N6GN/K over a 21 km line of sight path, the data and a best-fit Gamma distribution with Shape=5.62, Scale=0.00132. Next, in R use those parameters to generate a set of random (spectral width) values conforming to that Gamma distribution, Figure A1 (lower), using:

```
synth_n6gn_total<-rgamma(n=500, shape=5.62,
scale=0.00132)

fit_synth_n6gn_total<-fitdist(synth_n6gn_total, distr =
"gamma", method = "mle")

summary(fit_synth_n6gn_total)      # print summary statistics
including Shape and Scale

plot(fit_synth_n6gn_total)         # plot the distribution and
Gamma fit
```

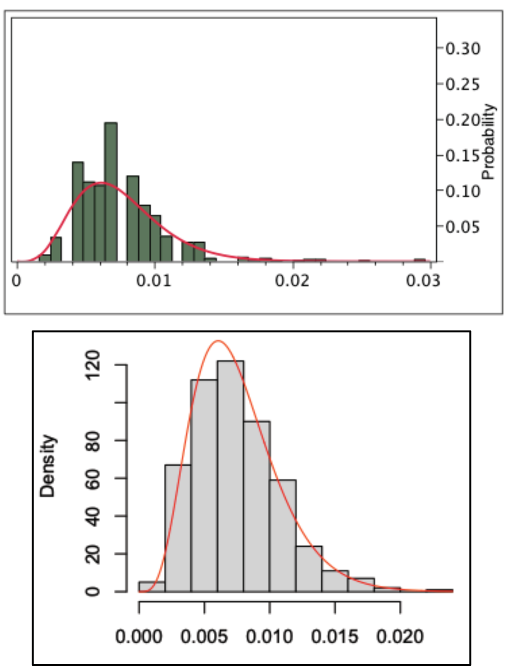

Figure A1. (upper) Histogram of actual spectral width values in Hz over a 21 km line-of-sight path from N6GN to N6GN/K with identical GPSDO master oscillators at the transmitter and receiver with a best-fit Gamma distribution. (lower) A synthesised histogram of a Gamma distribution of 500 values with the same Scale and Shape parameters as the observations.

2. Next, assume we can neglect any spectral spreading over this line-of-sight path. We further assume that the measured spectral spreading was equal at the transmitter and receiver; a reasonable assumption as the GPSDOs were identical. It is a property of Gamma distributions that if we halve the Shape parameter, keeping the same Scale parameter, and add the independent, individual random variables drawn from those two half-Shape distributions we end up with the original distribution. Therefore, we can synthesise individual GPSDO transmitter and receiver spectral spreads from:

```
n6gn_tx<-rgamma(n=500, shape=2.81, scale=0.00132)

n6gn_rx<-rgamma(n=500, shape=2.81, scale=0.00132)
```

These will be distributions of random spectral width values, that of the receiver shown in Figure A2 (upper). In Figure A2 (lower) we have added the individual values from the independent transmitter and receiver distributions and plotted using the commands in step 1. Note that individual columns differ slightly as different random values from distributions with the same parameters have been used. The Shape and Scale are also slightly different at 5.105 and 0.00146. Using more than 500 points would likely bring closer agreement.

```
synth_n6gn_tx_rx<-n6gn_tx+n6gn_rx
```

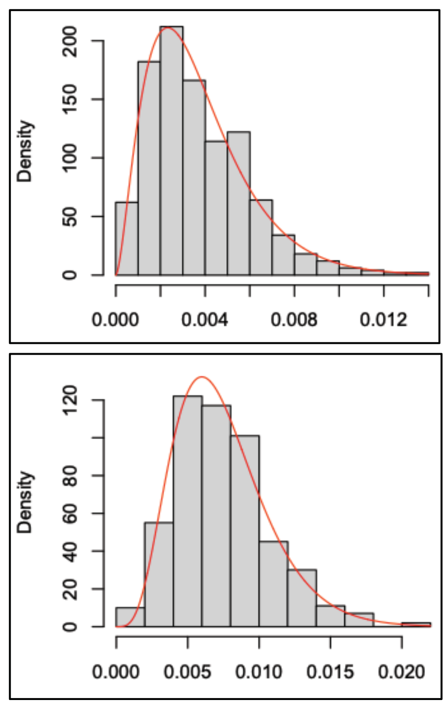

Figure A2. (upper) Histogram with fitted Gamma distribution for the N6GN/K GPSDO receiver only, formed by halving the Shape parameter from Figure A1 keeping the same Scale. (lower) Synthesised distribution for the transmitter and receiver to compare with Figure A1 (lower).

3. Next we take the measured spectral width distribution from the same transmitter, N6GN, to WB7ABP/K, with a GPSDO receiver, over a single-hop ionospheric path of 1558 km, Figure A3 (upper), fit a Gamma distribution

with Shape=1.878 and Scale=0.0215, and synthesise a distribution of 500 random values, Figure A3 (lower).

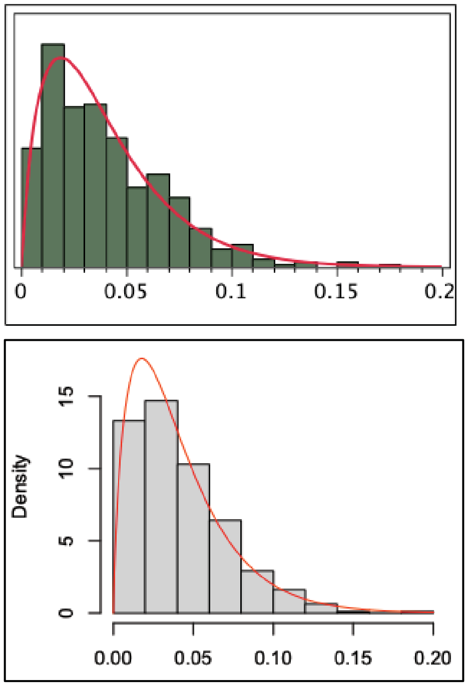

Figure A3. (upper) Histogram with fitted Gamma distribution for the total path from N6GN to WB7ABP/K including contributions from the transmitter and receiver. (lower) Synthesised distribution for the total path to compare with Figure A3 (upper).

4. To arrive at the propagation path only spectral spread we subtract the individual random values generated for n6gn_tx and n6gn_rx (as the equipment at WB7ABP was the same) from the individual random values forming the distribution in Figure A3 (lower). The absolute function is needed as the subtraction produces some small negative values. Next we fit a Gamma distribution to the path only set of values. The resulting distribution, Figure A4, has Shape=1.151 and Scale-0.0297.

```
path_n6gn_to_wb7abp<-abs(n6gn_at_wb7abp - n6gn_tx - n6gn_rx)

fit_path_n6gn_to_wb7abp<-fitdist(path_n6gn_to_wb7abp, distr = "gamma", method = "mle")
```

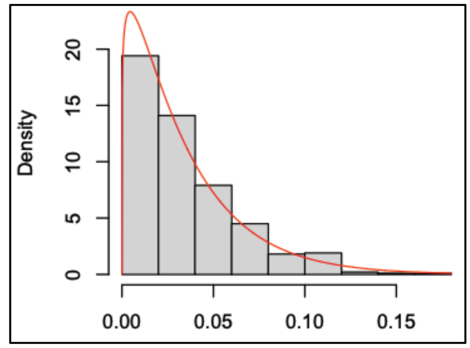

Figure A4. Synthesised distribution with fitted Gamma curve for the path-only spectral spreading from N6GN to WB7ABP/K.

5. From the Shape and the Scale parameters for the transmitter, receiver and path separately we calculate spectral spread values at a cumulative probability of 90% - that is, the value likely to not be exceeded 90% of the time. [This is easiest in Excel using gamma.inv (0.9, Shape, Scale)]. Those values are used as the individual component spectral spreads in Figure 8.
6. The same method is used for the other examples. The estimate for the KiwiSDR receiver with GPS aiding is made over the 38 km path from K6PZB to KPH knowing the K6PZB GPSDO transmitter spread. Knowing the KiwiSDR spread the method in step 4 is used for the two-hop path from N6GN to AI6VN/KH6, whose receiver is a standard GPS aided KiwiSDR.

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[1] Franke, S., Somerville, B. and Taylor, J., Quick-Start Guide to FST4 and FST4W, available at https://physics.princeton.edu/pulsar/k1jt/FST4_Quick_Start.pdf checked August 2022

[2] Website at http://wsprdaemon.org/ code available at https://github.com/rrobinett/wsprdaemon and real-time list of WsprDaemon listeners with FST4W shown as F* is at http://wsprdaemon.org/fst4w

[3] Testing of HF Modems with Bandwidths of up to about 12 kHz Using Ionospheric Channel Simulators, Recommendation ITU-R F.1487, International Telecommunications Union, 2000.

[4] Watterson, C.C., Juroshek, J. and Bensema, W., 1970. Experimental confirmation of an HF channel model. *IEEE Transactions on communication technology*, *18*(6), pp.792-803.

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[6] Angling, M.J., Cannon, P.S., Davies, N.C., Lundborg, B., Jodalen, V. and Moreland, K.W., 1995, September. Measurements of Doppler spread on high latitude HF paths. In Proceedings of the AGARD Sensor and Propagation Panel Symposium on Digital Communications Systems: Propagation Effects, Technical Solutions, Systems Designs.
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